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ON THE VARIATION AND INVERTIBILITY OF ROOM IMPULSE RESPONSE FUNCTIONS

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This paper presents results concerning variations in impulse response functions measured in different rooms. The variation of these functions for changing source and receiver positions and orientation was examined by assessing the change in energies of the direct and reflected signal components. The various source/receiver positions and orientation examined during these tests correspond to typical positions occupied by human speakers in rooms. It was found that the response functions changed drastically with the variation of these recording parameters. The feasibility of inverting (deconvolving) such functions by using digital signal processing techniques was also examined. It was found that when the exact response function was employed, then such an operation was feasible, leading to enhancement and reduction in the reverberant energy present in signals (e.g., speech) recorded under similar conditions. However, when a response recorded at a different position in the same enclosure was employed, then, in general, the inverse filtering operation increased the distortions present in the signals. This indicates that signal dereverberation by impulse response deconvolution has a limited scope in most practical situations.

1. INTRODUCTION

Acoustic enclosures can be modelled as linear systems whose transmission properties between source and receiver are described by an impulse response function, $h(t)$. In general such systems are time-invariant but, often, some parameters related to them may change between or during measurements. In practice, changes often occur due to variations in source and receiver (s/r) positions, and for directional transducers changes can occur due to variations in their orientations. In addition, other minor changes such as the position of physical objects in the room, the opening of doors, etc., may affect the system under consideration. When these minor effects and receiver directionality are ignored, a room response can be uniquely defined by a set of spatial co-ordinates, defined by a parameter vector l, corresponding to the three source position and three receiver position co-ordinates inside the room, together with a parameter vector $\theta(\theta_s, \phi_s, D)$ specifying the source relative polar and azimuth angles θ_s and ϕ_s in respect to the source-receiver axis, and source directivity, D (this being sufficient for axisymmetric source directivity). A discrete time representation of signals will now be adopted, according to which the signal $r_{l,\theta}(k)$ recorded at the receiver position inside the room due to an (anechoic) input signal $s(k)$, will be described as

$$r_{l,\theta}(k) = s(k) * h_{l,\theta}(k) = \sum_{j=0}^{k} s(j) h_{l,\theta}(k-j), \qquad (1)$$

where $h_{l,\theta}(k)$ defines the discrete time room impulse response function corresponding to the parameters l and θ, and j is an integer.

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The advantage of such a model is that it allows efficient simulation and assessment of room reverberation by using digital computers. Furthermore, as was shown in reference [1], deconvolution (inverse filtering) can be applied to reverberant signals in order to recover the original anechoic signal. However, such an operation is viable only when the record of the response is available prior to inversion. In the study presented here the practical feasibility of deconvolution is evaluated, primarily for speech enhancement applications, since speech is often degraded by room reverberation. Also discussed is the mismatch created when deconvolution is applied with an inexact parameter response (for s/r positions and orientation). Such mismatch can be created when the speaker is changing positions inside the room, so that effectively the room impulse response function in relation to a stationary receiver also changes. To fully evaluate such effects, a set of experiments was conducted, and the variation of response functions for changes in these parameters was measured with use of a number of established criteria. In this respect, the experimental results presented here correspond to the typical source/receiver configurations encountered for recording human speakers in rooms. A generalized discussion of the variation of sound fields in rooms for changing source/receiver positions was given by Doak [2].

The paper begins with a discussion of the experimental and computational procedure for the recording and analysis of real room impulse response functions. In the same section, aspects of the invertibility of such functions are also presented, and criteria to evaluate such operations are introduced. In the next section, a study of the variation of measured response functions is presented, followed by results referring to the invertibility of these responses.

2. METHOD

2.1. MEASUREMENT TECHNIQUE

The choice of the response measurement technique was dictated by the requirement to simulate the properties (frequency/dynamic range and directivity) of human speakers. The frequency and dynamic range requirements were successfully met by the swept sine excitation method, proposed by Berkhout *et al.* [3]. The directionality requirements were met by an Artificial Speech Source, consisting of a specially constructed loudspeaker enclosure. This arrangement gave directionality results (in the horizontal plane) comparable to those measured for human speakers [4].

For the tests, a 100-4000 Hz, 100 ms duration sweep excitation signal was employed, so that the measured responses were defined between those frequency limits that correspond to the perceptually significant speech bandwidth. This signal was generated digitally, and the responses were acquired at a 10 kHz sampling rate, by using the Data Analysis Centre DATS-11 computer system, at the I.S.V.R. The excitation signal was introduced in all rooms, at a preset level, corresponding to 70 dB *SPL* at 1 m under anechoic conditions. At this level the equipment was found to behave in a linear fashion.

The room response functions were obtained by inverse filtering to remove the excitation signal and equipment distortions, as shown in Figure 1. This method has also been described in reference [3]. The inverse filtering algorithm employed for the extraction of the impulse response functions was different from the one originally proposed in this reference. For the estimation of inverse filters, an optimum delay, least squares method was used, as described in reference [5] (see also Appendix). This technique produces finite length inverse filters whose length due to computational limitations is restricted to 1023 points.

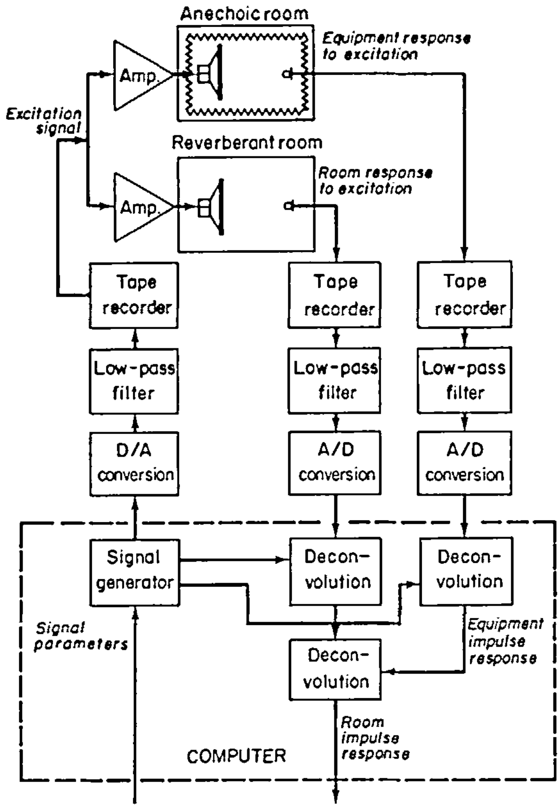

Figure 1. Schematic representation of room response measurement technique.

2.2 EXPERIMENTAL PROCEDURE AND CRITERIA

The study was conducted in 3 different enclosures, referred to as R1, R2, and R3. The physical and acoustical properties of these enclosures are given in Table 1.

TABLE 1

Description of enclosures employed for the tests

Description of enclosure	Dimensions (m)	Volume (m^3)	Reverberation time RT(s) (average 100–4000 Hz)	Critical distance r_{cd} (m) (corrected†)
R1 Small office	3×3·15×3·5	33·07	0·7	0·89
R2 Laboratory	8·9×6·65×4	235	1·55	1·56
R3 Reverberant chamber	—‡	348	5·1	1·05

† The corrected critical distance values are approximate results, in the sense that an average directivity value D (between 100 and 4000 Hz) is employed for the Artificial Speech Source. These critical distance values were estimated according to the expression $r_{cd} = \sqrt{D \times 0{\cdot}0032 \times V/RT}$.

‡ Enclosure R3 is not rectangular in shape.

For all tests, the source and receiver distance and orientation was varied by moving the receiver along each room's diagonal axis. The source was placed either towards the room centre or corner. Three different source orientation measurements (here defined by the horizontal angle between the loudspeaker axis and room diagonal) were taken for each source/receiver placement, for angles $\theta_1 = 0°$, $\theta_2 = 90°$ and $\theta_3 = 180°$. The height of both source and receiver was maintained at 1·5 m.

The responses were analyzed by using time domain criteria, mainly because such criteria can indicate the intelligibility of speech, recorded under similar conditions. For the first criterion it is considered that each measured response $h(k)$ consists of a direct (path) component, $d(k)$ and a reverberant signal component, $g(k)$: i.e.,

$$h(k) = d(k) + g(k). \tag{2}$$

When the first $k_0 - 1$ samples of $h(k)$ are assumed to correspond to this direct signal, then an energy ratio between these two components can be produced [6]. Hence the direct to reverberant energy criterion E_d/E_r is defined as

$$E_d/E_r = 10 \log \left\{ \sum_{k=0}^{k_0-1} d^2(k) / \sum_{k=k_0}^{\infty} g^2(k) \right\} \text{(dB)}. \tag{3}$$

Jetzt [6] has shown that this criterion also describes the (average) frequency domain transmission properties of enclosures. In practice, finite length estimates of E_r were considered, dictated by an upper time limit, k_{max}, imposed by the dynamic range of the recording equipment ($\simeq$40 dB). The interval k_{max} was chosen in such a way that the measured responses had decayed to the noise floor level of the equipment. Effectively, the recording equipment noise will bias the practical estimates of the energy ratio criterion. To minimize such effects, samples of $h(k)$ beyond k_{max} were removed by multiplication with a cosine-taper data window. Due to this operation, the E_d/E_r criterion was modified to

$$E_d/E_r = 10 \log \left\{ \sum_{k=0}^{k_0-1} d^2(k) / \sum_{k=k_0}^{k_{max}} g^2(k) \right\} \text{(dB)}. \tag{4}$$

The second criterion employed for the tests was chosen so that the beneficial to intelligibility early portion of the room response [7] was incorporated with the direct signal. This criterion, often refered to as the "early" to "late" energy ratio, is defined as

$$E_e/E_l = 10 \log \left\{ \sum_{k=0}^{k_{50}} h^2(k) / \sum_{k=k_{51}}^{k_{max}} h^2(k) \right\} \text{(dB)} \tag{5}$$

where k_{50} and k_{51} are sample numbers corresponding to 50 and 51 milliseconds of continuous time data.

It must be also noted that for the estimation of the above criteria from measured responses, any time delay due to the direct source/receiver path was removed, so that the direct signal component was always assumed to appear at $k = 0$.

2.3. THE INVERSION OF ROOM RESPONSE FUNCTIONS

As with other applications of deconvolution, inversion of room functions depends upon estimating a filter with response $h_{i,l,\theta}(k)$, such that

$$h_{l,\theta}(k) * h_{i,l,\theta}(k) = \delta(k), \tag{6}$$

where $\delta(k)$ is a delta function. In practice, only an approximate inversion can be obtained, producing $\hat{\delta}(k)$, so that an inversion mismatch error $e(k)$ is also produced, defined as

$$e(k)=\delta(k)-\hat{\delta}(k). \tag{7}$$

When $h_{i,l,\theta}(k)$ is employed to filter reverberant speech (see equation (1)), then, clearly, the smaller $e(k)$ is the greater is the success of the deverberation. Of particular importance to this study are errors arising from mismatch in the parameters employed for the estimation of such inverse filters. For example, it is often possible to employ a record of a response obtained at a position (co-ordinates) l' and orientation θ', instead of the original parameters l and θ in order to estimate the inverse filter. In addition to such errors, evaluated in section 3.2.2, other practical sources of error were found to occur during response inversion. These arise mainly from computational approximations in estimating the inverse filter. For assessment of such errors it is important to consider the mixed phase properties of room response functions [1], and to take into account that inverses for such functions must ideally be two sided. To avoid difficulties appearing in the estimation of such functions, Neely and Allen [1] suggested the following response decomposition:

$$h(k)=h_{eq}(k)*h_{ap}(k). \tag{8}$$

$h_{eq}(k)$ and $h_{ap}(k)$ denote the ("equivalent") minimum phase and all-pass response components [8]. They also suggested that an inverse to $h_{eq}(k)$ could be produced to remove the spectral distortions imposed by the room transmission properties. This effectively creates a mismatch error approximately equal to $h_{ap}(k)$, the room response all-pass component. Since this component was found [9] to carry most of the reverberant energy in the response, it is desirable to adopt a different approach to response inversion. Here, an alternative method is suggested, according to which a finite length, mixed phase inverse operator is designed by using an optimum delay least squares approach. Details of this method were presented in reference [5] (see also the Appendix), where the superiority of this method to a homomorphic filtering inversion method was also shown.

The mismatch error (given by equations (6) and (7)) will effectively determine the success of the dereverberation by response deconvolution, since it will appear as residual reverberation and distortion on the processed speech signal. This can be illustrated by observing that the reverberant signal after filtering (see equations (6) and (7)) will be

$$r_{l,\theta}(k)*h_{i,l,\theta}(k)=s(k)*\hat{\delta}(k)=\hat{s}(k), \tag{9}$$

where $\hat{s}(k)$ denotes an estimate of the original anechoic signal. To evaluate the residual distortions imposed on $s(k)$ by $\hat{\delta}(k)$ (see equation (9)), a modified E_d/E_r criterion was employed for the inversion operation, defined as

$$E_{di}/E_{ri}=10\log\left\{\mathrm{d}^2(k)/\sum_{k=0}^{k_{max}}e^2(k)\right\}\ (\mathrm{dB}), \tag{10}$$

where $e(k)=\delta(k)-\hat{\delta}(k)$ (from equation (7)).

An invertibility criterion, I, was then defined by comparison of the original response and the result of inversion, given by

$$I=E_d/E_r-E_{di}/E_{ri}. \tag{11}$$

Hence this value of I gives the reverberant energy (in dB) removed from the room response (or from any signal recorded at the same position) after response deconvolution.

3. RESULTS

3.1. THE VARIATION OF MEASURED RESPONSES

In this section the effects of source/receiver placement and orientation on the E_d/E_r and E_e/E_l criteria described earlier are considered. For the placement variation study, the source/receiver distance was altered so that typical measurements (in room R2) were taken at $l_1 = 1$ m, $l_2 = 1{\cdot}5$ m, $l_3 = 4$ m, and $l_4 = 9$ m.

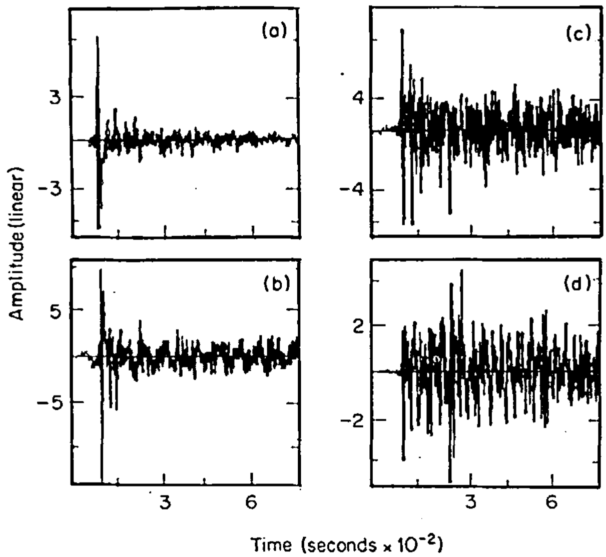

Figure 2. Impulse response functions, recorded in room R2 for increasing source receiver distance: (a) s/r distance = 1 m; (b) s/r distance = 1·5 m; (c) s/r distance = 4 m; (d) s/r distance = 9 m (initial response part is shown).

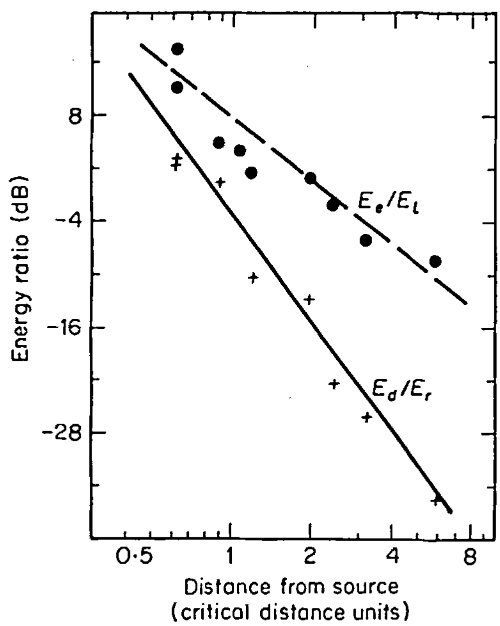

Figure 3. Effect of source/receiver distance on the response energy ratios. ——, Direct to reverberant ratio; - - -, early to late ratio.

Such results, illustrating the variation of the response with distance are shown in Figure 2. Results for all enclosures are plotted in Figure 3. From observation of these results it appears that both criteria (energy ratios) decrease at approximate rates of 12 and 8 dB respectively for each doubling of the source/receiver distance (expressed in critical distance units). These results indicate that the measured, average reduction in the direct : reverberant energy ratio with distance is significantly higher than the one expected from theory, when a uniform reverberant field is assumed [6]. It can be also observed that both criteria decrease faster (with source/receiver distance) in rooms with small volume and/or long reverberation than in rooms with large volume and/or short reverberation. It is useful to note that speech intelligibility will also decrease in a similar fashion with distance due to the reduction in the direct signal component [10].

The above results correspond to direct axis measurements ($\theta = 0°$), in the three enclosures.

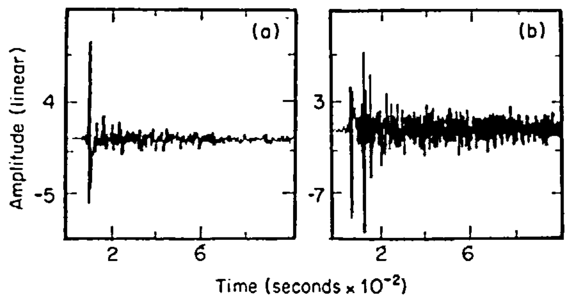

Figure 4. Impulse response functions, recorded in room R2 for varying source/receiver orientation: (a) s/r orientation = 0°; (b) s/r orientation = 90° (initial response part is shown).

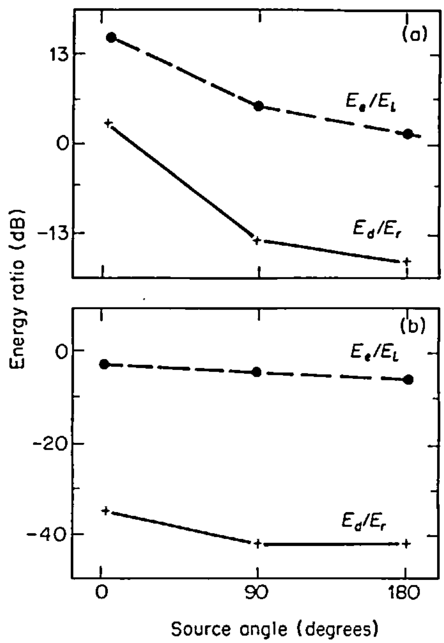

Figure 5. Effect of source/receiver orientation on the response energy ratios: (a) s/r distance = 1 m; (b) s/r distance = 9 m. ——, Direct to reverberant ratio; - - -, early to late ratio.

In the study of response variation with s/r orientation it has been assumed that, due to the directionality properties of the Artificial Speech Source, the results will correspond to the variation observed for human speakers, speaking at different angles towards the receiver.

The discussion presented here refers to measurements taken in room R2, although comparable results were also obtained in the other rooms. Figure 4 shows the strong variation of measured responses with changing s/r orientation. As is shown in Figure 5, both E_d/E_r and E_e/E_l criteria were found to decrease for all the non-direct axis measurements. For the near s/r positions, the reduction in the E_d/E_r ratio was approximately 15 dB for a 90° variation and a further 5 dB for the 180° orientation. For the longest source/receiver position, the measured reduction in both criteria was smaller (8 and 9 dB respectively), indicating the diminishing importance of the direct signal component with distance. Speech intelligibility will, in general, decrease in accordance with the reduction in the direct (or "early") signal component, as can be observed in many everyday situations when a speaker moves and/or turns away from the listener.

3.2. THE INVERTIBILITY OF MEASURED RESPONSES

3.2.1. *Exact response inversion*

As was discussed earlier, the occurrence of mismatch error (according to equation (7)) for the exact response inversion case is related to the computational approximations restricting the length of the inverse filter employed for deconvolution. Nevertheless, the response inversion succeeds in removing most of the reverberant energy from this signal, as can be observed from Figure 6. In Figure 7, the invertibility measure, I, as described by equation (11), is plotted against s/r distance and original degradation. Clearly, the effect of inversion is more beneficial for the longest s/r positions and for the lower E_d/E_r ratio cases. A similar improvement must be expected when reverberant signals are processed in this way as illustrated by equation (9). The quantitative effects after the deconvolution of such signals are described by the response invertibility criterion, I, whose dependence on various parameters is given by Figure 7. The qualitative effects of

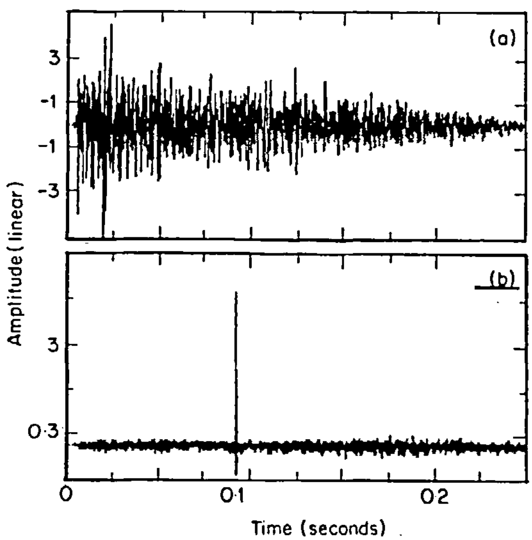

Figure 6. Inversion results for a room impulse response: (a) original response recorded in room R2, for s/r distance = 9 m; (b) result of inverse filtering applied on this function.

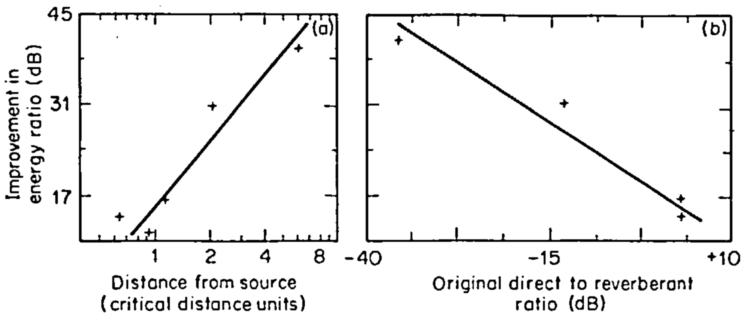

Figure 7. Effect of inverse filtering when the exact impulse response functions are used: (a) energy ratio improvement for the different s/r distance recordings; (b) energy ratio improvements *vs.* the original energy ratio.

inversion were also evaluated, by using informal listening tests to assess the subjective effects of processing on reverberant speech.

For these tests, the speech sentences were recorded under anechoic conditions and were introduced into the reverberant room via the Artificial Speech Source. For the same s/r parameters, the room response was also measured (as was shown in Figure 1) and its inverse was estimated and employed to filter the reverberant speech. A schematic representation of this operation is given by Figure 8.

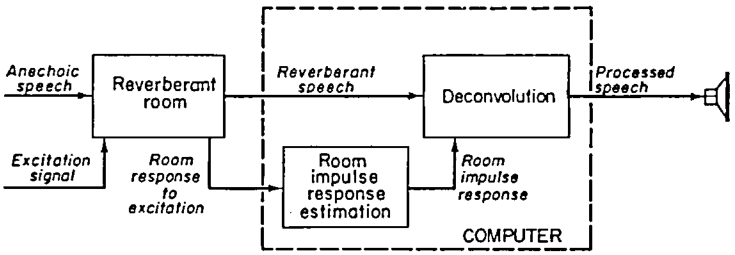

Figure 8. Set-up for the speech processing scheme to evaluate the effect of response inversion.

From these tests it was found that the processed speech was characterized by a reinforcement of the direct (path) speech component, as is also indicated by the improvement in the invertibility criterion, *I*. However, the processed signal was not completely free from reverberation; significantly, frequency domain distortions (coloration) were not removed, and in fact, they were often reinforced after processing. Such effects were more prominent in the longest s/r distance measurements and can be attributed to the inability of the relatively short inverse filter to remove the fine spectral irregularities that are imposed by the room on the speech signal. Such spectral irregularities are perceived as the characteristic reverberant coloration effect, and as was shown by Jetzt [6] they are more prominent for the longest s/r distance recordings (i.e., when the direct signal component level is lower).

3.2.2. *Inexact response inversion*

Given the variation of room response functions, described in section 3.1, any inversion in which incorrect parameter responses are used will generate considerable mismatch error. Here, for simplicity, only the mismatch error created by the variation of s/r positions is to be discussed. For this study, inverse filters were employed that were created from responses measured at increasing error distance from the actual response position. The invertibility criterion, I (as defined by equation (11)), was applied in all cases, defined here as

$$I=[E_d/E_r]_{l,\theta}-10\log\{\delta^2(k)/\sum_{k=0}^{k_{max}} e_m^2(k)\}\ (\text{dB}), \tag{12}$$

where $e_m(k)=\delta(k)-[h_{l,\theta}(k)*h_{i,l',\theta}(k)]$.

Table 2 gives the relationship between I and the error distance $|l-l'|$ for the actual and the assumed positions, expressed by (corrected) critical distance units. These results refer mainly to room R2 and indicate that, in general, a degradation (increase of distortions) must be expected when inexact response inverse filters are employed for deconvolution. This degradation appears to increase with error distance, although it was also found that the results were strongly dependent on the particular recording and room geometric configuration.

Similar degradation was also observed (see Table 2), when a simulated impulse response was employed for deconvolution. For this test, a synthetic impulse response was generated by using the image approach algorithm suggested by Allen [11]. The simulation parameters

TABLE 2

Results of room response invertibility for inexact parameters

Room and position	Error distance, $\lvert l-l'\rvert$ (m)	(Corrected critical distance units)	Invertibility, I† (dB)
R2, position 1	0	0	+18·55
R2, position 2	0·5	0·32	−1·4
R2, position 3	5·0	3·2	−39·07
R2, simulation (see text)	0	0	−4·22
R3, position 1	0·8	1·0	−10·29

† Positive values of I indicate improvement and negative values indicate degradation.

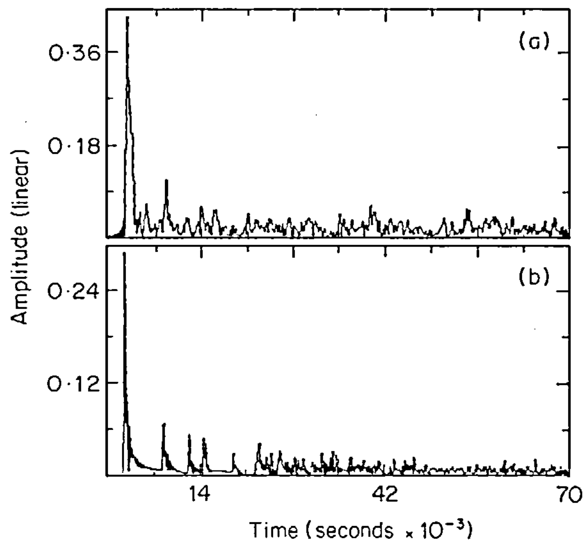

Figure 9. Comparison between the envelope functions for the (a) original recorded response and (b) result of computer simulation, according to the method suggested in reference [11].

were dictated by the dimensions of room R2 and the exact s/r positions where the actual response was measured. However it must also be noted that for the simulation an omnidirectional source is assumed (see Figure 9).

The inexact response inversion results indicate that deconvolution with such responses will, in general, degrade signals. For example, an average 10 dB degradation in the E_d/E_r criterion must be expected for each doubling of the error distance (normalized to critical distance units). The results of section 3.1 also indicate that a further degradation will be generated for inexact estimates of the s/r orientation. The results from the limited number of tests conducted also indicate that in order to obtain a consistent improvement from inversion (in the E_d/E_r ratio) the response used must be measured no further apart than 1/2 critical distance units from the actual recording position. (This distance corresponds to approximately 0·35 m in an average size room of $RT=1$ s.)

Clearly, such a condition cannot be satisfied in many practical situations, indicating the limited scope for reverberant signal enhancement by response deconvolution methods.

4. CONCLUDING REMARKS

The study of impulse variation for three different rooms showed that these functions can vary drastically with changes in source and receiver positions and orientation. Such variations were evaluated from the broad response properties: i.e., the energy ratios between the direct (or early) and reverberant (or late) part of these functions. Variation was also observed in the detailed structure of measured response functions: i.e., due to changes in amplitude and arrival intervals for the individual wall reflections. Although such effects were not properly evaluated, it is believed that they mainly create the mismatch error observed when deconvolution is carried out with use of inexact responses. The invertibility study also indicated that such error increases with the error distance between the actual and the assumed response functions. In practice this limitation restricts speech dereverberation by estimated response deconvolution, since even small errors in the response parameters will further degrade the speech signal.

In conclusion, the short study described here indicates that a unique linear system representation for a room holds true only for each specific source/receiver position and orientation. It must also be noted that comparable changes are not usually observed (perceived) in the acoustic quality of an enclosure for changing source and listener positions.

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APPENDIX: THE INVERSION OF MIXED PHASE SIGNALS

As room impulse responses are mixed phase functions, then in general they are not directly invertible by using the usual discrete Fourier transform (D.F.T.) methods [8]. For this, alternative inversion methods have to be employed.

The principles of least squares inversion rely upon designing a causal and finite duration filter $h_i(k)$ such that when convolved with the signal $h(k)$ it produces the best least squares approximation to a delta function. As $h(k)$ is a mixed phase signal, then the output delta function has to be delayed appropriately, in order to compensate for the finite energy build-up time in $h(k)$. If p denotes this delay, then in order to obtain $h_i(k)$ it is required to minimise a function I, of the form

$$I = \sum_{k=0}^{M=p} [\delta(k-p) - y(k)]^2,$$

where $y(k) = h(k) * h_i(k)$ and M denotes the duration of $h(k)$. As was discussed in reference [5], the solution for the optimal filter reduces to a set of normal equations that can be solved by using the Levinson recursion algorithm. In addition, the delay p can be estimated by incorporating the efficient "Simpson sideways recursion" algorithm [5]. Further details can also be found in reference [12].

An alternative method for inverting mixed phase signals was also given in reference [5]. This technique relies upon cepstrally separating $h(k)$ into a maximum and minimum phase component (see also reference [9]). Unlike $h(k)$, these two components can be inverted by D.F.T. techniques. In general, any minimum phase function has a stable and causal inverse, and a maximum phase function has an acausal and stable inverse. In practice, the inverse operator for this function has to be rendered causal after the introduction of an appropriate delay. Finally, a composite inverse filter $h_i(k)$ can be obtained from the convolution of the two separate component inverses.