The ECG Signal Decomposition Using IIR Wavelet Filter Banks
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I. INTRODUCTION
In this paper we examine the orthogonal wavelet decomposition of ECG signal obtained using computationally efficient IIR perfect reconstruction quadrature-mirror filter (QMF) banks. The decomposition is done using the lowpass branch iterated filter banks. The fifth order Butterworth and elliptic filters are employed. IIR filters’ polyphase components are two real all-pass structures of the first order. The ortonormality property is attained by block processing implementation of anticausal filtering in the synthesis part of IIR banks. We compare these results with the decomposition obtained by the seventh order Daubechies 4 FIR filters.

II. WAVELET FILTER BANKS
Figure 1 shows the amplitude responses of lowpass filters used in wavelet decomposition. The Butterworth and elliptic filters of the fifth order are more selective than the seventh order Daubechies 4 filter. The Butterworth and elliptic filters belong to an important class of IIR filters which can be realized using allpass filters as polyphase components, [1]. The main Butterworth filter feature is maximal magnitude response flatness at Nyquist frequency, while the elliptic filter of the same order has the best frequency selectivity. Both IIR filters have better frequency selectivity than the higher order FIR filter.

\[ h_1(n) = (-1)^n h_0(N-1-n), \quad n = 0,\ldots,N-1 \] (1)
where N denotes the filter order

The impulse responses of the synthesis bank filters are time-reversed versions of the analysis filters, so perfect reconstruction property is satisfied. In the case of the FIR filter banks this means an appropriate filter coefficients time – inversion.

![Figure 1](image1.png)

**Figure 1** Gain responses of the half-band lowpass filters

Let, \( H_0(z) \) and \( H_1(z) \) denote the transfer functions of the lowpass and highpass filter of the analysis part of the two-channel QMF bank, then Figure 2 illustrates analysis and synthesis blocks of orthonormal perfect reconstruction two-band filter bank, [2]. Orthonormality condition imposes the connection between the lowpass and highpass filter coefficients. The coefficients of the highpass analysis filter, \( h_1(n) \), are obtained by reversing in order and alternating in sign the lowpass analysis filter coefficients, \( h_0(n) \), i.e. the highpass coefficients are

![Figure 2](image2.png)

**Figure 2** Analysis and synthesis filter bank blocks

The wavelet banks are generated using the same two-channel filter bank after the decimated lowpass filtered signal of the previous filter bank, [3]. Six level wavelet decomposition block scheme, utilizing Daubechies 4 the seventh order FIR filters, is depicted in figure 3. The wavelet detail coefficients are denoted as \( d_1, d_2, d_3, d_4, d_5, d_6 \) and the approximation coefficient as \( a_6 \). In order to provide system to have the perfect reconstruction property, the decimated highpass filtered signal denoted by \( d_5 \) has to be delayed by \( K \) samples. \( K \) is overall delay of two-channel FIR filter bank shown in figure 2. Signal denoted by \( d_4 \) has to be delayed by \( 3-K \) samples, since the previous inserted delay, between the downsampling and upsampling by a factor 2 i.e. multiplied by 2, is enlarged by delay of the preceding filter bank. In the similar way the other inserted delays are computed.

![Figure 3](image3.png)

**Figure 3** Six-level FIR filter wavelet bank
A computationally efficient polyphase realization of two-channel IIR QMF bank is depicted in Figure 4. The IIR filters are of the fifth order. The polyphase components are the first order all-pass filters \(A_0(z)\) and \(A_1(z)\). The filters \(A_0(z)\) and \(A_1(z)\) can be then written as:

\[
A_0(z) = \frac{a_0 + z^{-1}}{1 + a_0 z^{-1}}
\]

\[
A_1(z) = \frac{a_1 + z^{-1}}{1 + a_1 z^{-1}}
\]

The corresponding synthesis two-band filter bank is shown in the bottom part of the same figure. Since an all-pass filter, \(A_{0,1}(z)\), has the property \(A_{0,1}(z)A_{0,1}(z^{-1}) = 1\), it is obvious the two-band analysis/synthesis structure has the perfect reconstruction property. In the synthesis bank polyphase components are anticausal filters, [4]. In this way a linear phase response is obtained. Since the anticausal filters are unstable systems, a method for causal implementation of the anticausal filters should be used. In this paper, the causal implementation of the anticausal filter is based on the corresponding causal filter and the block processing technique, [5, 6]. The block processing technique is a direct procedure for the design with the smallest length of the time-reversed sequence and with a small processing delay and can be used for the processing of the infinite length sequences or very long finite input sequences.

![Figure 4 Analysis and synthesis blocks of the polyphase realization of two-channel IIR QMF bank](image)

The six level wavelet block scheme, utilizing the fifth order IIR filters, is depicted in figure 5. There is no need for delays in highpass branches because of sequence inversion in block processing technique. The wavelet detail coefficients are denoted as \(d_1, d_2, d_3, d_4, d_5, d_6\) and approximation as \(a_6\).

![Figure 5 Six-level IIR filter wavelet bank](image)

**III. WAVELET ECG DECOMPOSITION**

We use QMF two-band filter banks for decomposition of an ECG test sequence. Detail of the ECG test sequence is

![Figure 6 The ECG signal](image)

![Figure 7 Daubechies 4 wavelet decomposition of the ECG signal](image)

![Figure 8 The ECG signal wavelet decomposition using filter banks of the fifth order Butterworth filters](image)

![Figure 9 The ECG wavelet coefficients computed using filter banks of the fifth order elliptic filters](image)
illustrated in figure 6. Wavelet coefficients are computed using the filter bank structures explained previously. Wavelet coefficients obtained using Daubechies 4 filter banks, the fifth order Butterworth filter banks and the fifth order elliptic filter banks are depicted in figures 7, 8 and 9, respectively. The corresponding coefficients, obtained with different filter banks, are very similar.

IV. WAVELET RECONSTRUCTION

Wavelet decomposition is of a great importance especially in subband coding of ECG signal, [7]. The quality of reconstruction is also significant in this application. The ECG signal can be processed in a batch mode or in blocks of suitable size, so the reconstruction of the ECG signal is done using block processing technique. We compare reconstructed signals for different block lengths. Figure 10 illustrates reconstructed ECG signals for the Butterworth and elliptic filter case. The reconstructed signals are the same as the original signal depicted in figure 6. This reconstruction is done for block processing length of \( L = 10 \). To investigate further the effect of block length on reconstruction quality, we observe distortions for block lengths of \( L = 10, 15, 20, 25 \) when the input signal has triangle shape. The triangle shape input signal can be viewed as a simplified detail of ECG pattern. The test signal is 51 sample long and its amplitude equals 1.

Figures 11 and 12 illustrate reconstruction distortions when filter banks of Butterworth filters are applied for block lengths \( L = 10, 15, 20, 25 \). Distortions are noticeable only on flat part of the output signal before the triangle rising edge. The horizontal part of the output signals are magnified and shown in figure 12. The distortion amplitude decreases rapidly as the block length increases.

Figures 13 and 14 depict reconstruction of the test signal when elliptic filters are used. The block lengths take values \( L = 10, 15, 20, 25 \). The conclusions are the same as in the Butterworth case. The lengthier block makes smaller distortions in reconstructed signal.

Comparing figures 12 and 14, we conclude that Butterworth filter wavelet bank has smaller distortions of the reconstructed signal than the elliptic filter wavelet bank for the same block processing length. For instance, for block length \( L = 25 \) the distortion amplitude is \( 10^{-7} \) for Butterworth filters and \( 10^{-4} \) for elliptic filters.
Daubechies 4 FIR filter of the seventh order have eight distinct coefficients, taking into account the relationship between coefficient of the lowpass and the highpass filter, equation (1), analysis can be realized with eight multiplications per one output sample. Since the six stage wavelet decomposition is observed, the overall number of multiplication per output sample is 48.

In the IIR case, employed filters are of the fifth order and are realized using parallel connection of two first order allpass sections. This means that the corresponding difference equation for polyphase filters can be implemented with one single multiplication. The one stage analysis can be done using only 2 multiplications per output sample. The complete sixth order decomposition is done with 12 multiplications per input sample.

With respect to computational complexity the IIR wavelet decomposition needs four times less multiplication per output sample than the Daubechies 4 FIR filters analysis.

V CONCLUSIONS

In this paper, the ECG signal wavelet decomposition, using the fifth order IIR filter banks, is examined. The fifth order Butterworth and elliptic filters are employed. The results are compared with the decomposition obtained using Daubechies 4 seventh order FIR filter banks. The IIR filters have better selectivity properties than inspected the higher order FIR filters. The IIR filters are also computationally more efficient as they utilize four times less multiplications per output sample than the FIR filters.

ACKNOWLEDGEMENT

The authors are grateful to Prof. Dr. Miodrag Popović for helpful discussions.

REFERENCES


